

Algebraic geometry 2

Exercise Sheet 5

PD Dr. Maksim Zhykhovich

Summer Semester 2026, 13.05.2026

Exercise 1. Let A be an integral domain. Denote by $\text{Frac } A$ the fraction field of A . Let ξ be the point of $X = \text{Spec } A$ corresponding to the prime ideal (0) .

(1) Recall that $\mathcal{O}_{X,\xi} \simeq \text{Frac } A$.

(2) Show that the structure sheaf $\mathcal{O}_X$ on X is a subsheaf of the constant sheaf $\mathcal{F}$, $U \mapsto \text{Frac } A$.

Remark: Since X is connected, $\mathcal{F}$ is indeed a sheaf.

Hint: Observe that ξ lies in every open $U \subset X$ and, thus, we have a ring homomorphism $\mathcal{O}_X(U) \rightarrow \mathcal{O}_{X,\xi}$.

Exercise 2. (1) Let X be a scheme. Show that X is reduced if and only if the stalk $\mathcal{O}_{X,P}$ is a reduced ring (has no nilpotent elements) for every $P \in X$.

(2) Let A be a ring. Show that $\text{Spec } A$ is a reduced scheme if and only if A is a reduced ring.

Exercise 3. Let $X = \text{Spec } A$ be an affine scheme.

(1) Show that X is irreducible if and only if $\text{Nil } A$ is a prime ideal.

(2) Show that X is integral if and only if A is an integral domain.

Exercise 4. Find an example of an affine non-integral scheme $X = \text{Spec } A$ whose all stalks $\mathcal{O}_{X,P}$ are integral domains.